are real and positive, but may be fractional or integral.

While tables of the zeros of (1) do exist for certain ranges of p and k (see [1], [2], [3]) these are in general rather limited in extent, particularly for large and fractional values of p and k. Hence, for the solution of problems involving non-tabulated values of the parameters it is necessary either to interpolate existing tabulations, or to solve (1) numerically. In many such instances much of the labor may be avoided by the use of a simple relation, derived by the author in the course of developing an approximate theory of propagation in rectangular waveguide wound into a helical form [for the exact treatment of this problem see [4], which also contains extensive tables of the zeros of (1)]. On the basis of the assumptions

a) that only the TE_{10} mode is propagated,
b) that electrical lengths may be measured along the axis of the waveguide, and
c) that the pitch of the helix is negligible (see [4] for justification of this),

the following formula for the roots of (1) is derived:

$$x_0 \doteq \sqrt{\frac{\pi^2}{(k-1)^2} + \frac{4p^2}{(k+1)^2}}\,. \qquad (2)$$

The values x_0 obtained from (2) are very close approximations to the first zeros of (1). The closeness of the approximation depends upon the particular values of k and p under consideration, and for a given case may be estimated by reference to the accompanying Fig. 1. If the point determined by (k, p) lies within the central cross-hatched region, the resultant value of x_0 will in general be within ± 1 per cent of the exact value, though if it lies within the region bounded by the dashed curve the lower limit may drop to -1.5 per cent. If the point lies anywhere within the diagonally-hatched region the value of x_0 calculated from (2) will be within ± 5 per cent of the exact value.

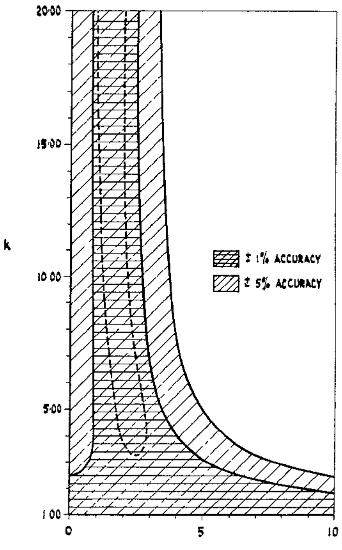

Fig. 1.

For the design engineer, accuracies within ± 1 per cent will often be adequate, and in such cases the use of (2) obviates the need for interpolation of tables or other tedious calculation. In other cases, where high accuracy is required, the use of (2) will quicly provide an excellent "first guess" which will permit a rapidly convergent numerical solution of (1). It may further be noted that, given any two of the three parameters p, k, x_0, the third may readily be calculated from (2) and the accuracy of the result determined from Fig. 1.

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Maximum Efficiency of a Two Arm Waveguide Junction*

It is well-known that the efficiency of a two-arm waveguide junction (2-port) depends upon the reflection coefficient Γ_L of the load with which one of the arms is terminated. The efficiency is known to vary between the limits 0 and η_m (maximum efficiency) as Γ_L assumes all possible values within the unit circle. However, there seems to be no published analysis from which one can determine the particular Γ_L giving maximum efficiency if the characteristics of the waveguide junction are known.

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It can be shown[1] that the reflection coefficient Γ_M to give maximum efficiency can be calculated from

$$\Gamma_M = S_{22}^* + \frac{(1-S_{22}\Gamma_M)^* S_{11}S_{12}^*S_{21} + |S_{12}S_{21}|^2\Gamma_M}{(1-|S_{11}|^2)(1-S_{22}\Gamma_M) - S_{11}^*S_{12}S_{21}\Gamma_M}, \qquad (1)$$

where the asterisk * denotes the complex conjugate, the S-terms denote the scattering coefficients of the waveguide junction, and the load of reflection coefficient Γ_M terminates arm 2. The solution of (1) for Γ_M may be written

$$\Gamma_M = \frac{B}{2A}\left[1 \pm \sqrt{1 - \left(\frac{2|A|}{B}\right)^2}\right].$$

Where

$$A = S_{22} + S_{11}^*(S_{12}S_{21} - S_{11}S_{22}), \qquad (2)$$

and

$$B = 1 - |S_{11}|^2 + |S_{22}|^2 - |S_{12}S_{21} - S_{11}S_{22}|^2$$

in some cases, it is necessary to choose the algebraic sign in (2) to yield a value of Γ_M within the unit circle.

One Γ_M has been determined, the maximum efficiency η_M can be determined from the equation

$$\eta_M = \frac{Z_{01}}{Z_{02}} \cdot \frac{|S_{21}|^2(1-|\Gamma_M|^2)}{|1-S_{22}\Gamma_M|^2 - |(S_{12}S_{21}-S_{11}S_{22})\Gamma_M + S_{11}|^2}, \qquad (3)$$

where Z_{01} and Z_{02} are the characteristic impedances of arms 1 and 2, respectively, of the 2-arm waveguide junction.

It can be further shown[2] that the quantity A_I, the intrinsic attenuation (equivalent to the intrinsic insertion loss of Tomiyasu[3]) is given by

$$A_I = 10 \log_{10} \frac{1}{\eta_M}\,. \qquad (4)$$

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[1] A convenient way to show this, is to postulate lossless tuners attached to both arms of the waveguide junction and adjusted for maximum power to the load. Under this condition, a conjugate match is obtained at each terminal surface in each waveguide lead. For simplicity, one may assume a non-reflecting generator and load without significant loss in generality. A straightforward analysis then leads to the stated result.

[2] This is shown in a paper entitled "Intrinsic Attenuation" which is in preparation by the correspondent.

[3] Kiyo Tomiyasu, "Intrinsic Insertion Loss of a Mismatched Microwave Network," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp 40–44; January 1955.